NONLINEAR REGRESSION

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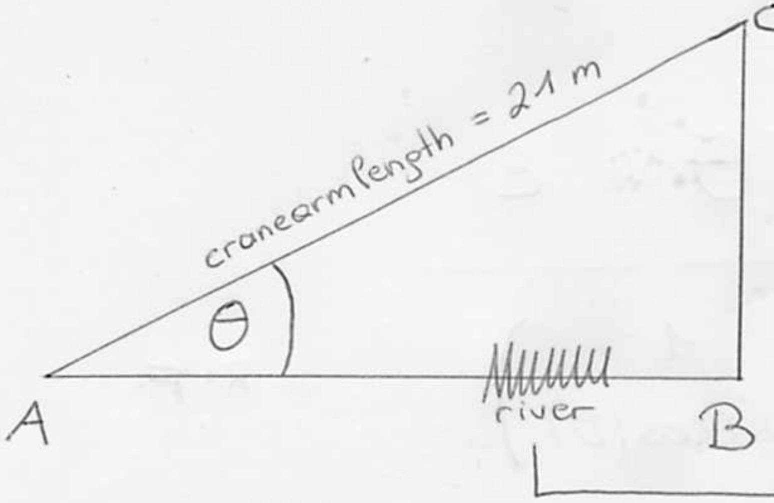
CRANE & PROTRACTOR & RIVER



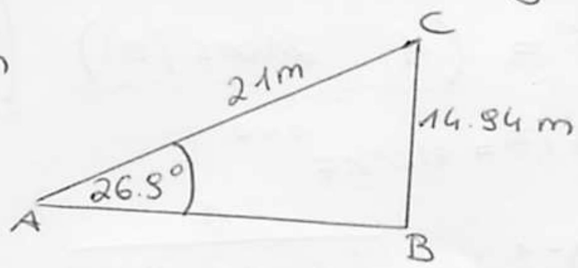
not possible to measure $\overline{AB}$

measurements:

in R:

for θ : $y_1 = 0.47$ rad $= 26.82902$ deg = theta

for $\overline{BC}$: $y_2 = 14.94$ m



$\boxed{1.}$ Create a Model:

$$\underline{Y = \eta(\theta) + \varepsilon} \qquad \theta \in \Theta$$

assumptions:
$Var(\varepsilon) = \sigma^2$
$\varepsilon \ldots$ i.i.d.

$$\eta(\theta) = \begin{pmatrix} \theta \\ 21\sin(\theta) \end{pmatrix} \qquad y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 26.82° \\ 14.94m \end{pmatrix}$$ in R: y

$$\Rightarrow \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} \theta \\ 21\sin(\theta) \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

2. Linear Approximation :

$$y^* = y - \eta(\theta^0) + \frac{\delta\eta(\theta^0)}{\partial\theta'}\theta^0$$

$$y^* = F\theta + \varepsilon$$

$$F = \frac{\partial\eta(\theta^0)}{\partial\theta'} = \begin{pmatrix} 1 \\ 21\cos(\theta) \end{pmatrix}$$ in R: F

$$F' = \begin{pmatrix} 1 & 18.72293 \end{pmatrix}$$ in R: Ft = t(F)

$$F'F = \underbrace{\begin{pmatrix} 1 & 21\cos(\theta) \end{pmatrix}}_{1\times 2} \underbrace{\begin{pmatrix} 1 \\ 21\cos(\theta) \end{pmatrix}}_{2\times 1} = \underbrace{\begin{pmatrix} 1 + 21^2\cos^2(\theta) \end{pmatrix}}_{1\times 1}$$

in R: FtF = Ft%*%F

$$(F'F)^{-1} = \underbrace{\frac{1}{(1 + 21^2\cos^2(\theta))}}_{1\times 1}$$ in R: FtFinv = FtF^(-1)

$$y^{*0} = \begin{pmatrix} 26.82° \\ 14.84m \end{pmatrix} - \underbrace{\begin{pmatrix} 26.82° \\ 21\sin(26.82) \end{pmatrix}} + \begin{pmatrix} 1 \\ 21\cos(26.82) \end{pmatrix} \cdot \underbrace{26.82}_{\text{deg(theta)}}$$

in R: ystar <- y - etatheta + F · theta

$$y^* = \begin{pmatrix} 0.47 \\ 14.22817 \end{pmatrix}$$

3. Various Prior Guesses / Starting Points :

$\theta^{\circ} = ?$ based on first measurement :

$\theta_1^{\circ} := y_1 = 26.82^{\circ} \hat{=} 0.47$ rad

based on second measurement :

$\theta_2^{\circ} := \arcsin\left(\frac{y_2}{21}\right) = 0.781529$

based on intuition :

$\theta_3^{\circ} := 30^{\circ} \hat{=} 0.5235888$ rad

4. Compute the Estimate $\hat{\theta}^*$

based on linear approximation :

$\hat{\theta}^* = (F'F)^{-1} F' y^*$ in R: thetaRatstar <- FtFinv %*% (Ft %*% ystar)

$\hat{\theta}_1^* = 0.7591611 \hat{=} 43.48673^{\circ}$

$\theta_3^* = 0.7668886 \hat{=} 43.94584^{\circ}$

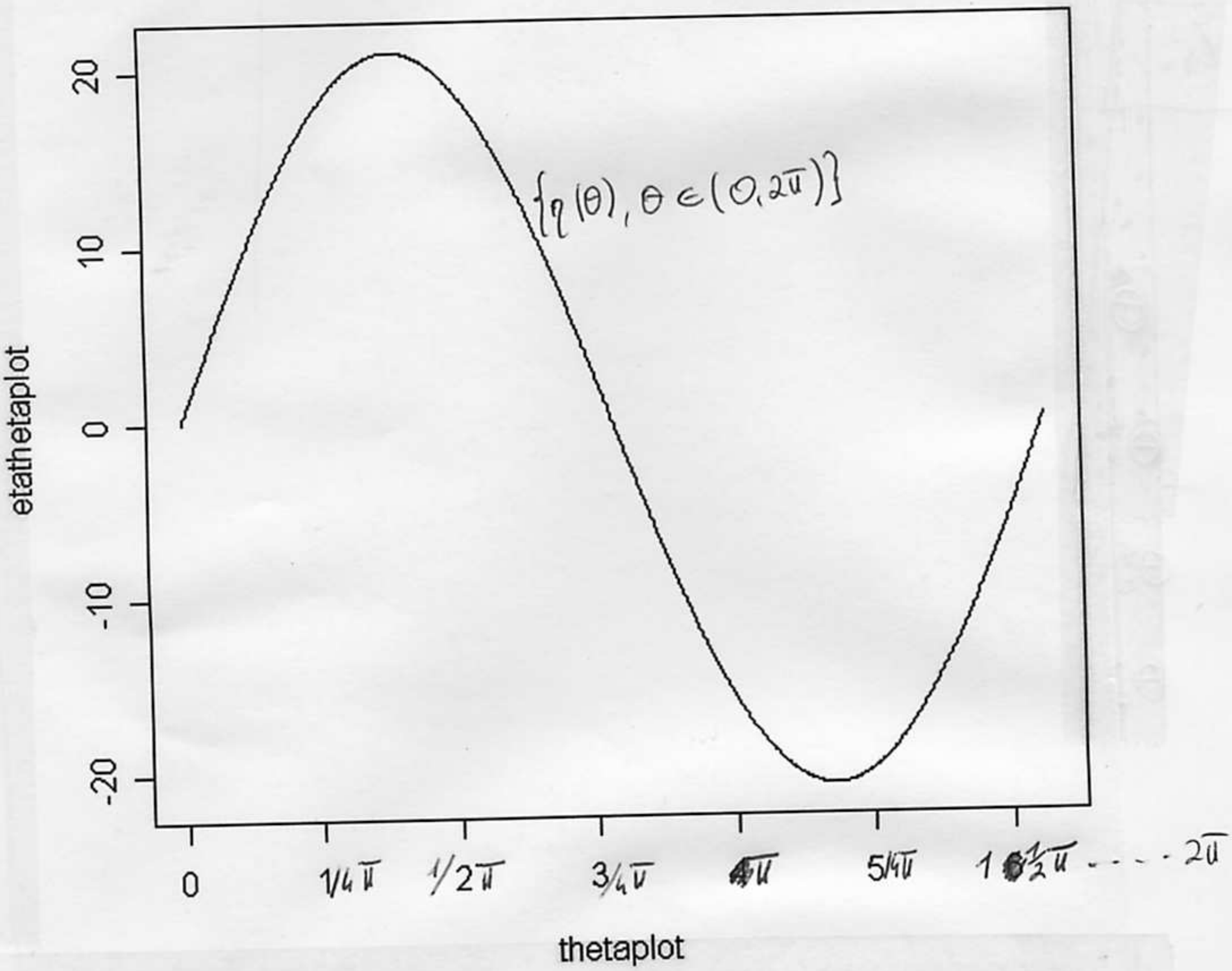
$\theta_2^* = 0.781529 \hat{=} 45.35127^{\circ}$

$\boxed{5.}$ Draw a picture of the Expectation Curve $\{\eta(\theta), \theta \in (0, 2\pi)\}$

in R:

```
thetaplot <- seq (0, 2*pi, 0.01)
etathetaplot <- 21* sin (thetaplot)
plot(thetaplot, etathetaplot, type = "p")
```

plot(tetaplot, etat



$\boxed{6.}$ Compute K_{int} (intrinsic curvature):

$$K_{int}(\theta) = \frac{\left\| I - P(\theta)\, \frac{d^2 \eta(\theta)}{d\theta^2} \right\|}{\left\| \frac{d\eta(\theta)}{d\theta} \right\|^2}$$

$$\underset{2\times1}{P(\theta) = F}\,(\underset{1\times1}{F'F})^{-1}\underset{1\times2}{F'}$$ in R: P <- F %*% FtFinv %*% Ft

$$\underset{2\times2}{P} = \begin{pmatrix} 0.004570459 & 0.0674505 \\ 0.06745050 & 0.9954295 \end{pmatrix}$$

$$\eta(\theta) = \begin{pmatrix} \theta \\ 21\sin(\theta) \end{pmatrix} \qquad \frac{d\eta(\theta)}{d\theta} = \begin{pmatrix} 1 \\ 21\cos(\theta) \end{pmatrix}$$

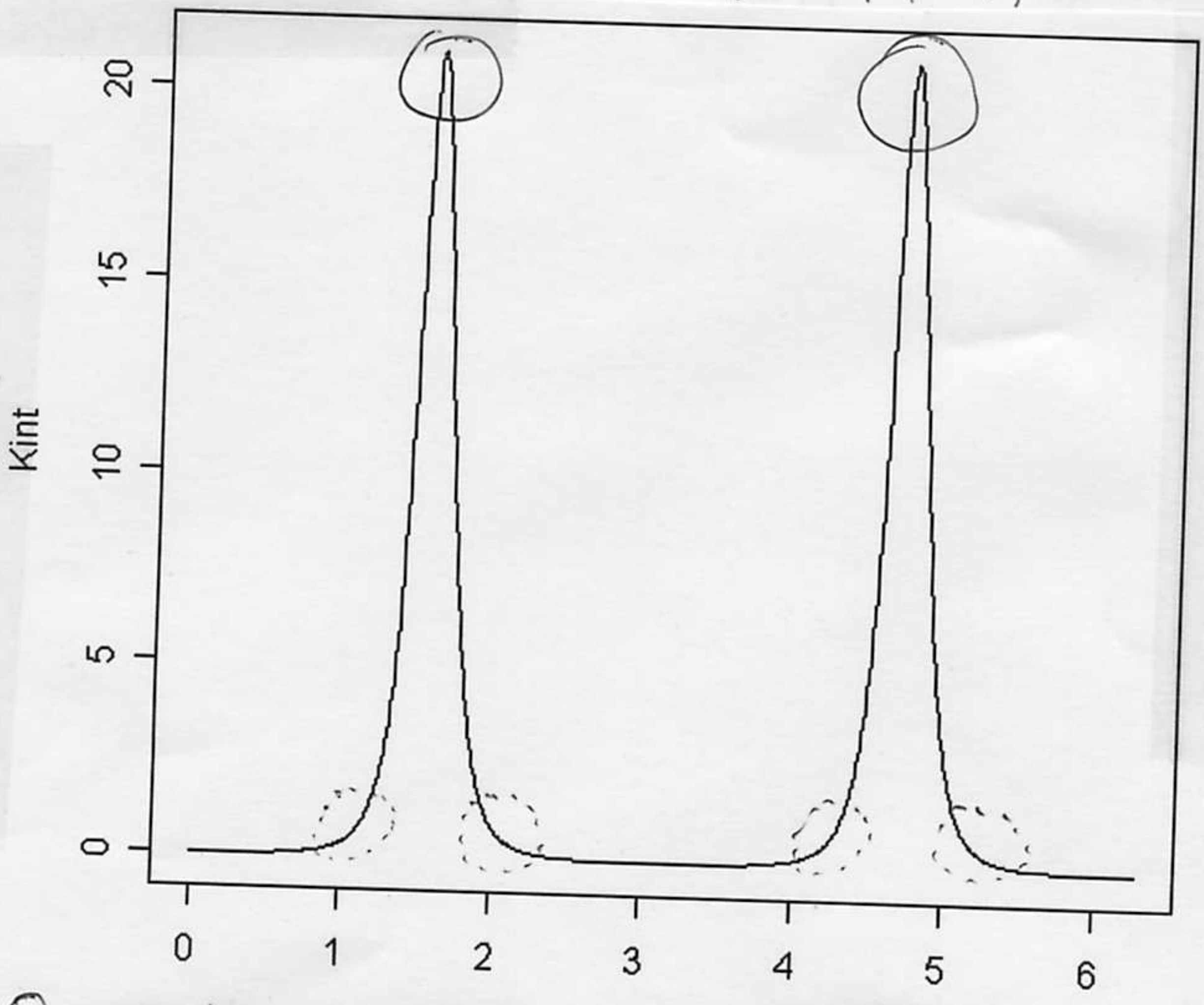
$$\frac{d^2\eta(\theta)}{d\theta^2} = \begin{pmatrix} 0 \\ -21\sin(\theta) \end{pmatrix}$$

in R:

```
izaehler <- 21*abs(sin(theta))/sqrt(1+
(21*cos (theta)*cos(theta)))
inenner <- sqrt((1+ (21*cos(theta)*cos(theta)))^3)
Kint = izaehler / inenner
```

$$\underline{\underline{K_{int}(\theta) = 0.03038202}}$$

Picture for Kind : $(\theta \in (0, 2\pi))$



O ... areas where curvation is at maximum ⇒ lin. approximation very bad at $(0, 2\pi)$ these points (inaccurate!)

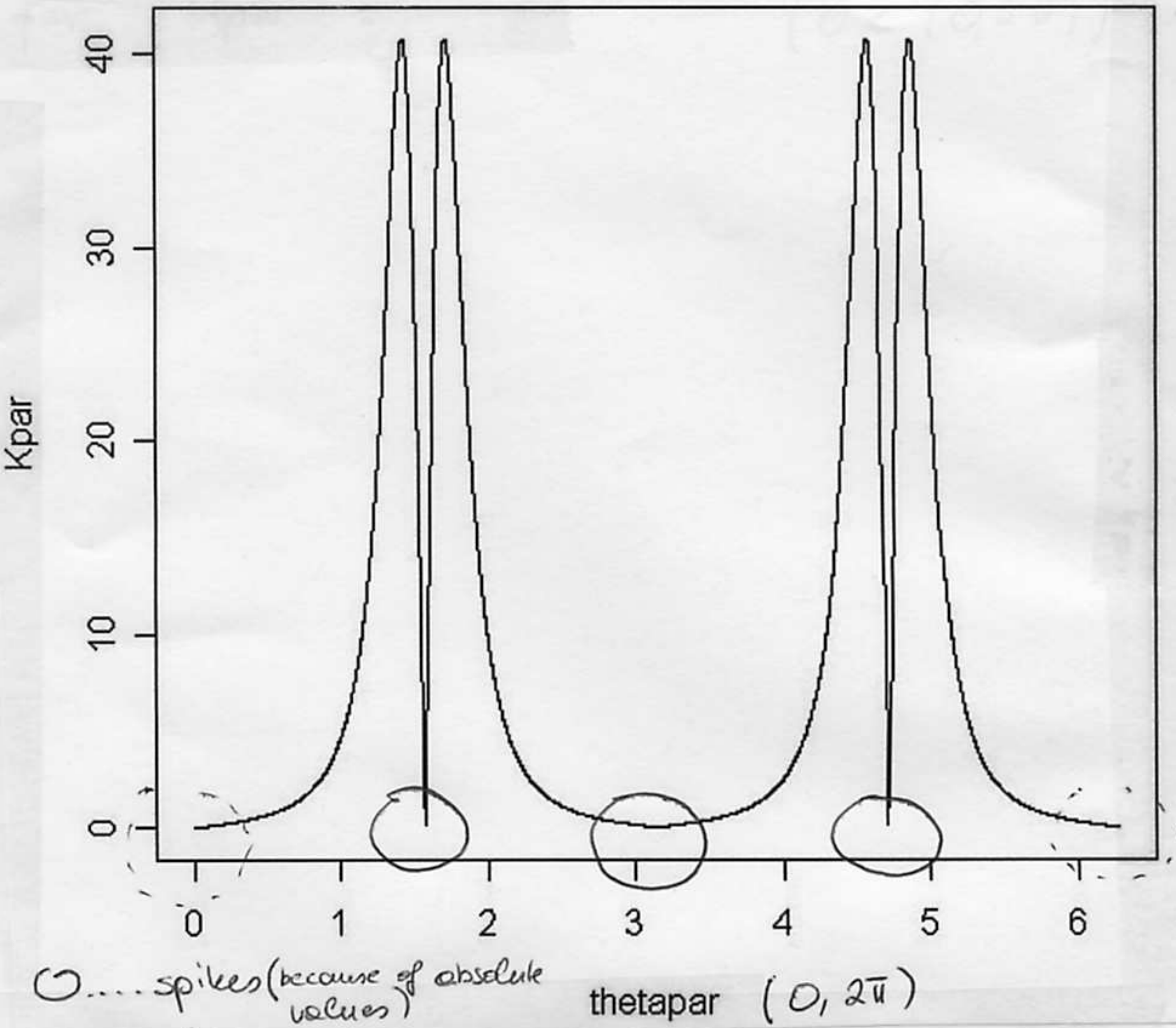
$\boxed{7.}$ Compute K_{par} (parameter effect curvature):

$$K_{par} = \frac{\left\| P(\theta) \cdot \frac{d^2 \eta(\theta)}{d\theta^2} \right\|}{\left\| \frac{d\eta(\theta)}{d\theta} \right\|^2}$$

in R:
```
pzaehler = 21^2 * abs(cos(theta)) * abs(sin(theta)) /
           (sqrt(1+(21 cos(theta) * cos(theta))))
pnenner = 1 + ((21 * cos(theta) * cos(theta))^(3/2)
Kpar = pzaehler/pnenner
```

$K_{par}(\theta) = 0.6117444$

Picture for Kpar:



O.... spikes (because of absolute values) $(0, 2\pi)$

$\boxed{8.}$ Interpretation:

K_{int} is relatively small => good situation because K_{int} gives us (1/) the radius of the circle that "snuggles" to the expectation curve at θ => the bending of the curve is not so strong and therefore a lin. approximation is quite fine at θ

For Kper is > 0 changing θ_i uniformly DOES NOT mean that the points $\eta(\theta_i)$ also change uniformly $\hat{=}$ the distances between these points are NOT equal // see also $\boxed{5.}$

9. Autofahrt:

Kpar can be also interpreted as absolute acceleration where + means 'Gas'
− means 'Bremse'

when you imagine driving ON the expectation curve as long as Kpar is > 0 the distances are changing (either in- or decreasing); if Kpar in-/decreases the change of distance-change is changing / the intensity of the change changes

have again a look at the pictures:

* --- you neither push the "Gaspedal" nor the "Bremse"

⊗ -- point where you fully push the pedal

→ see extra-page!

see extra - page

$\boxed{10.}$ Compute $\hat{\theta}$ as LS-estimate NOT biased on Linear approximation

$$\hat{\theta} = \underset{\theta \in \Theta}{\operatorname{argmin}}\, S(\theta)$$

$$S(\theta) = \| y - \eta(\theta) \|^2 =$$

$$= \left\| \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} - \begin{pmatrix} \theta \\ 21 \sin(\theta) \end{pmatrix} \right\|^2 \quad \text{für } \theta \in (0, 2\pi)$$

in R: theta <- seq(0, 2*pi, 0.1)

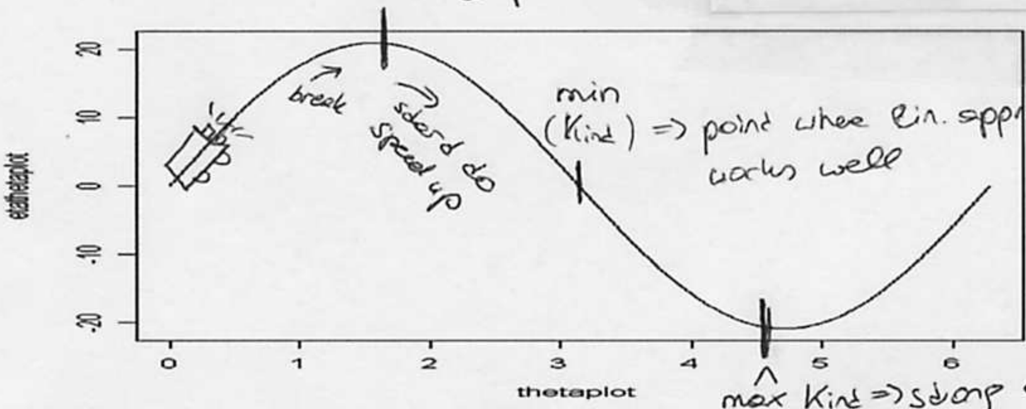
S <- sqrt((y[1] - theta)^2 + (y[2] - 21 *
 * sin(theta))^2)

min(S) = 0.3526964 at $\theta = 0.8\pi$

max. Kind => strong curve

Expecdation

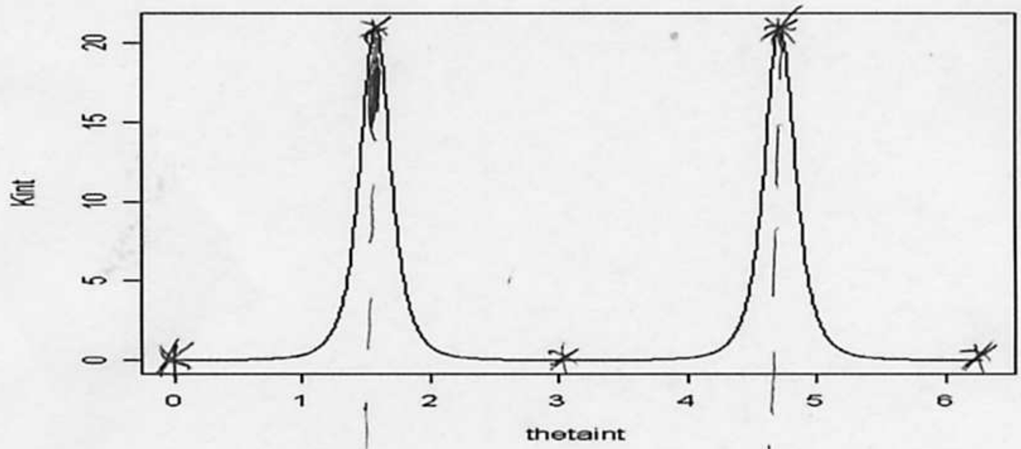
curve



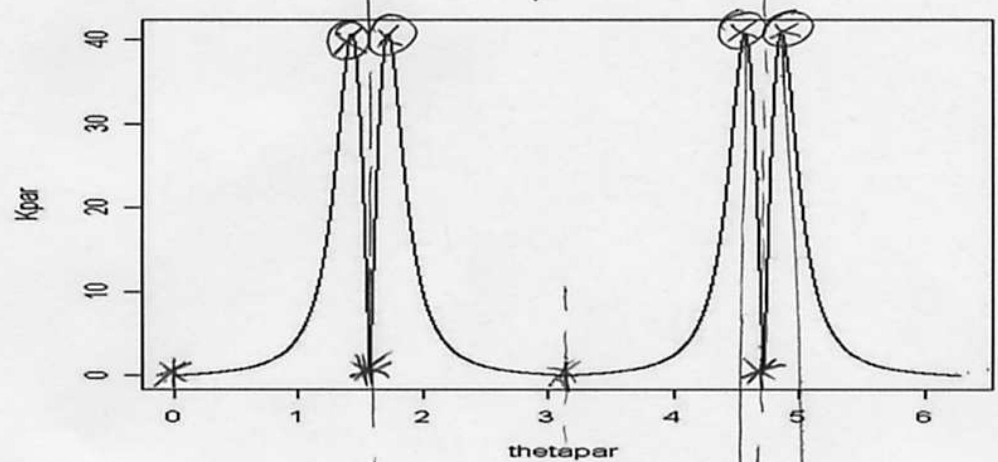
max Kind => strong curve

(X) point where you fully push the pedal and then you loosen the pedal => speed increases but increases slowlier now

Kind



Kpar



(X) (X)

GAS BREMSE

distances increase . . . .

distances decrease $(\eta(\theta_i)-\eta(\theta_{i+1}))$