Crane, Protractor and River

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1 Model

$$\begin{aligned} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} &= \begin{pmatrix} \theta \\ 21 \cdot sin(\theta) \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \\ y &= \eta(\theta) + \epsilon \end{aligned}$$

$y_1 = 0.47$ rad, $y_2 = 14.94$ m

2 Linear Approximation

$$\begin{aligned} y^* &= y - \eta(\theta^{(0)}) + \frac{d\eta(\theta^{(0)})}{d\theta} \cdot \theta^{(0)} \\ F &= \frac{d\eta(\theta^{(0)})}{d\theta} = \begin{pmatrix} 1 \\ 21 \cdot cos(\theta) \end{pmatrix} \\ \widehat{\theta}^* &= (F'F)^{-1}F'y^* \end{aligned}$$

Starting Point 1:

$$\theta^{(0,1)} = y_1 = 0.47$$

Result:

$$y^{*(1)} = \begin{pmatrix} 0.47000 \\ 14.22917 \end{pmatrix}, \theta^{*(1)} = 0.7591611$$

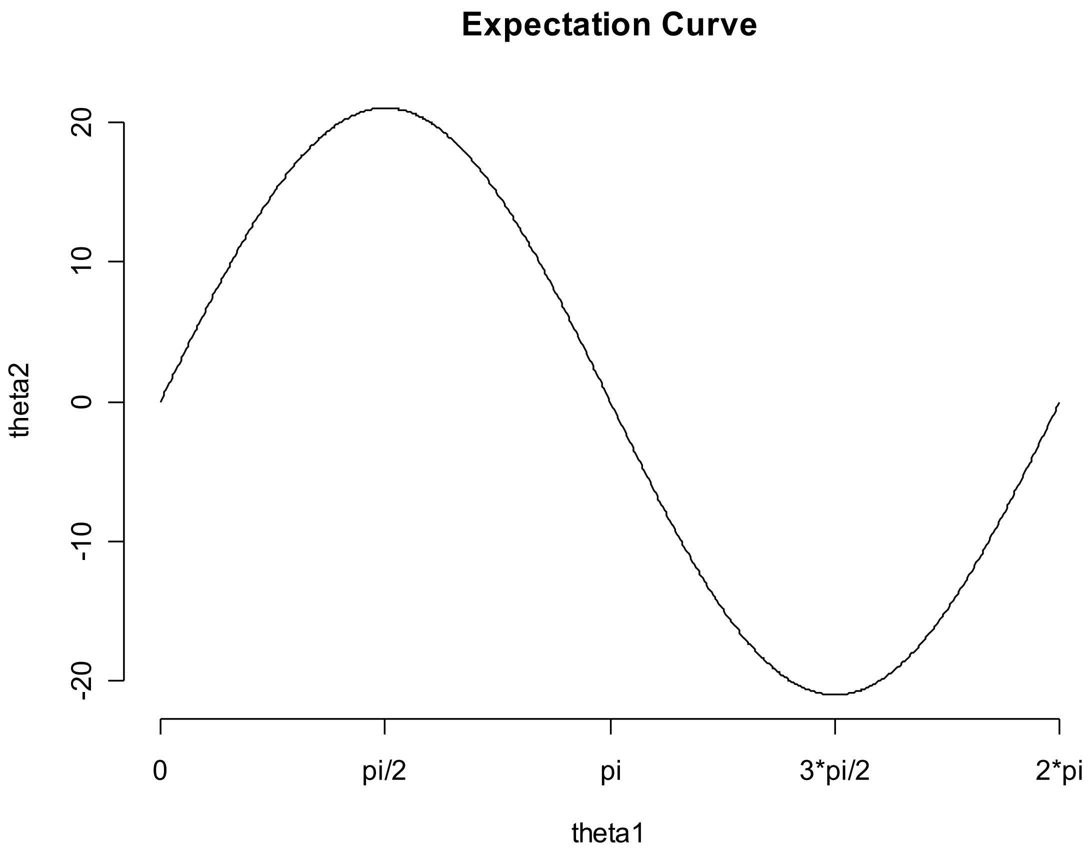
Starting Point 2:

$$\theta^{(0,2)} = arcsin\left(\frac{y_2}{21}\right) = arcsin\left(\frac{14.94}{21}\right)$$

Result:

$$y^{*(2)} = \begin{pmatrix} 0.47000 \\ 11.68133 \end{pmatrix}, \theta^{*(2)} = 0.7900594$$

3 Expectation Curve



4 Curvatures

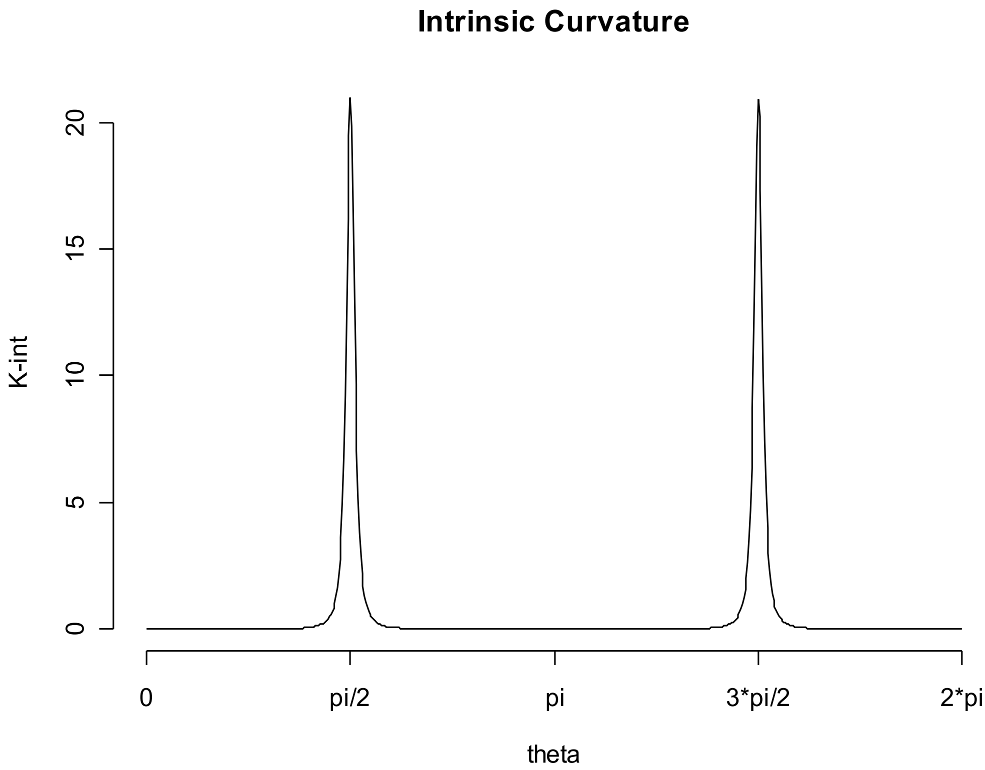
4.1 Intrinsic Curvature

$$K_{int}(\theta) = \frac{\left\| (I - P(\theta)) \cdot \frac{d^2\eta(\theta)}{d\theta^2} \right\|}{\left\| \frac{d\eta(\theta)}{d\theta} \right\|^2}$$

When the intrinsic curvature is small, the expectation surface is almost linear, so the linear approximation is quite accurate.

$$K_{int}(\theta^{*1}) = 0.00406246$$
$$K_{int}(\theta^{*2}) = 0.004589152$$

As can be seen on the following graph, these values are very small, so the linear approximation is good.



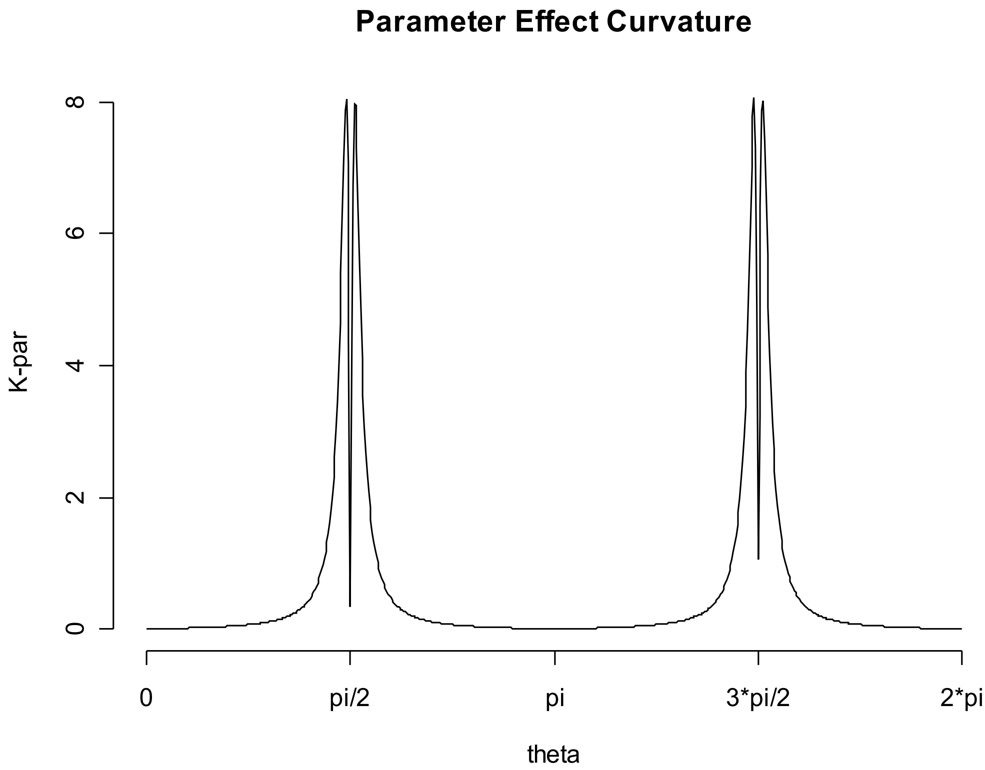
4.2 Parameter Effect Curvature

$$K_{par}(\theta) = \frac{\left\| P(\theta) \cdot \frac{d^2\eta(\theta)}{d\theta^2} \right\|}{\left\| \frac{d\eta(\theta)}{d\theta} \right\|^2}$$

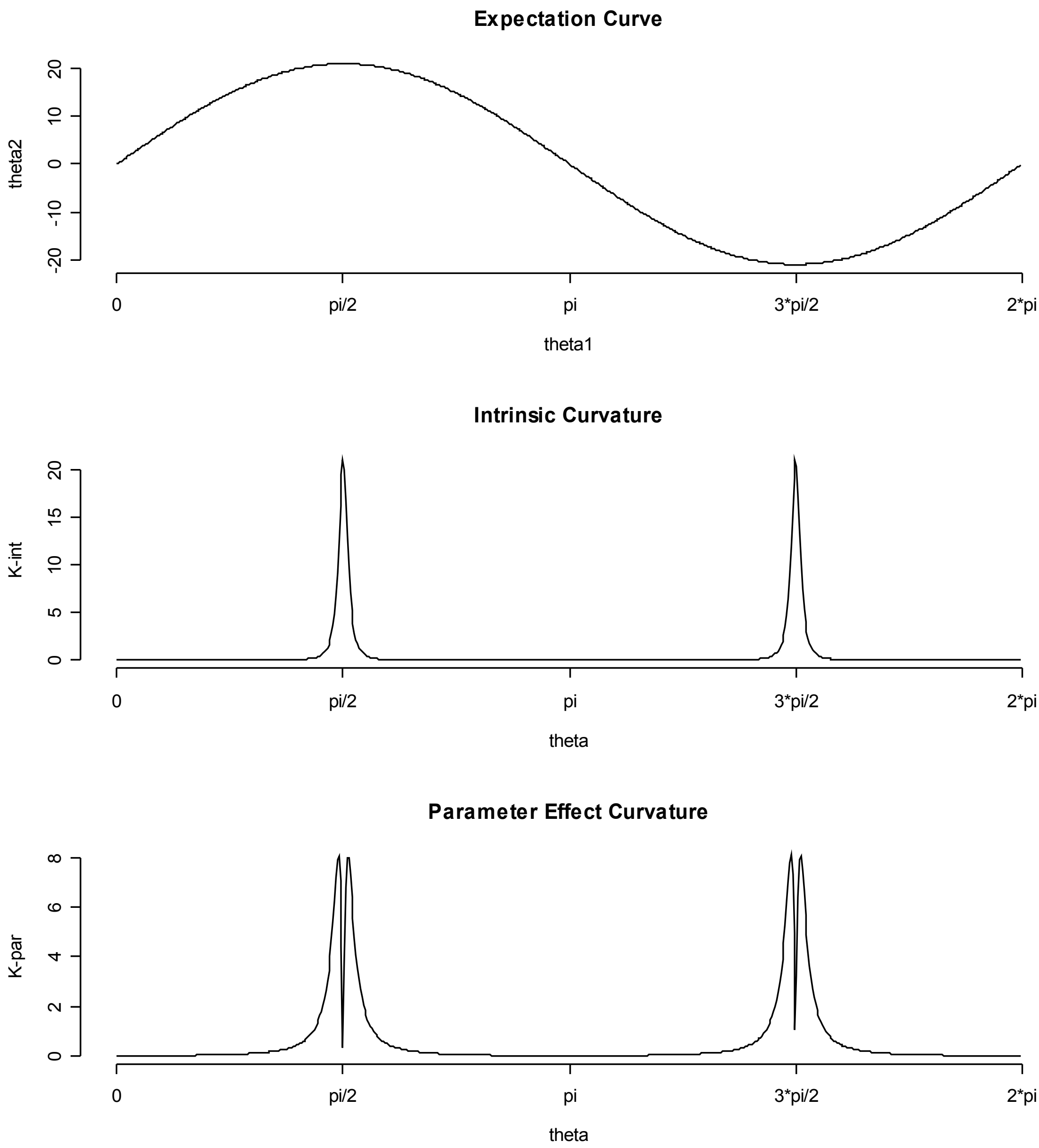
The parameter effect curvature expresses how changes in θ affect $\eta(\theta)$. If $K_{par} = 0$, uniform changes in θ lead to uniform changes in $\eta(\theta)$. If $K_{par} < 0$, the changes in $\eta(\theta)$ are changing.

$$K_{par}(\theta^{*1}) = 0.06188624$$
$$K_{par}(\theta^{*2}) = 0.06782704$$

Again both values are very small. That means that at both estimates constant changes in θ cause nearly constant changes in $\eta(\theta)$.


Parameter Effect Curvature
K-par
0
2
4
6
8
0
pi/2
pi
3*pi/2
2*pi
theta

5 Interpretation



At $\pi/2$ and $3 \cdot \pi/2$ there are the maximum and minimum of the expectation curve. At these points, there are also the maxima of the intrinsic curvature: There the linear approximation is accurate only very close the starting point. Whereas in the area around π, at the inflexion point of the expectation curve, the intrinsic curvature is zero. Here the expectation curve is nearly linear, so the linear approximation fits very well, even far away from the starting point. At 0 and $2 \cdot \pi$ there are also inflexion points which have the same properties.

The parameter effect curvature curve has its peaks shortly before and shortly after the optima of the expectation curve. At the optima and the inflexion points it is zero. That means that if θ changes constantly ($\theta_i + c = \theta_{i+1}$), the change from $\eta(\theta_i)$ to $\eta(\theta_{i+1})$ is constant too at the optima and inflexion points of the expectation curve.

When the parameter effect curvature increases, the distances between $\eta(\theta_i)$ and $\eta(\theta_{i+1})$ are increasing, for example in the interval from 0 to about 1.5 (first peak). When it decreases, the distances are decreasing to, as can be seen in the interval from about 1.6 (second peak) to π.

The parameter effect curvature can be interpreted as the strength, with which the gas or brake pedal has to be pushed, to drive the path of the expectation curve. Before a turn like the maximum the brake pedal has to be pushed increasingly hard. The point where it has to be pushed hardest is the peak of the parameter effect curvature curve, which is located shortly before the maximum. Then it can be released until the maximum, where no pedal is pushed. From then on the gas pedal is pushed with increasing strength just until the second peak of the parameter effect curve, afterwards strength is decreasing until the inflexion point of the expectation curve. Here the pedal is changed again. To drive around the minimum the procedure starts from the beginning.

The parameter effect curve only shows how hard the pedal is pushed, but not which one is currently in use.